# MATH 121

## Midterm Practice Questions

(1) True or False.

a) Let $W = \{f(x) \in P(\mathbb{R}) | f(x) = f(-x)\}$, where $P(\mathbb{R})$ is the set of all polynomials with real coefficients. ($P(\mathbb{R})$ is infinite-dimensional.) Then $W$ together with the common addition and scalar multiplication is a vector space over $\mathbb{R}$.

b) Let $V$ be a vector space. Let $A_1, A_2$ be two subspaces of $V$, and let $\gamma_1, \gamma_2$ be bases of $A_2, A_2$, respectively. Then $\gamma_1 \bigcap \gamma_2$ is a basis for $A_1 \bigcap A_2$.

c) If $f$ and $g$ are polynomials of degree $n$ then $f + g$ has degree $n$.

d) If $x$ is a nonzero vector in $V$ and $a, b \in F$, then $ax = bx$ implies $a = b$.

e) If $S$ is a subset that spans $V$ and $T, T' : V \to W$ are linear transformations such that $T(s) = T'(s)$ for all $s \in S$, then $T = T'$.

f) If $S$ and $S'$ are linearly dependent subsets of $V$ then $S \bigcup S'$ is linearly dependent.

g) The set $\{(x, y) | 5x - y + 1 = 0\}$ is a subspace of $\mathbb{R}^2$.

(2) Prove that the composition of two surjective functions is surjective.

(3) Let $T$ be a linear operator on a vector space $V$. Let $\lambda_1, ..., \lambda_k$ be distinct eigenvalues of $T$. If $v_1, ..., v_k$ are eigenvectors of $T$ such that $\lambda_i$ corresponds to $v_i$, then $\{v_1, ..., v_k\}$ are linearly independent.

(4) Let $V$ be a 3-dimensional vector space with basis $\{v_1, v_2, v_3\}$ and $W$ be a 2-dimensional vector space with basis $\{w_1, w_2\}$. Let $T : V \to W$ be the linear operator which expressed as a matrix in terms of these bases is

$$M = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

a) Find a basis for the kernel of $T$.

b) Find a basis for the image of $T$.

c) State the Rank-Nullity Theorem.

d) Are your answers for a) and b) consistent with the Rank-Nullity Theorem?

(5) Let $P_n(\mathbb{R})$ denote the vector space of all polynomials of degree $\leq n$ over $\mathbb{R}$. Let $F$ be the linear transformation

$$F : P_n(\mathbb{R}) \to P_{n-1}(\mathbb{R}), u \to u'.$$

Consider the basis $\alpha := \{1, ..., t^n\}$ for $P_n(\mathbb{R})$ and the basis $\beta := \{1, ..., t^{n-1}\}$ for $P_{n-1}(\mathbb{R})$. Compute the matrix for $F$ with respect to the bases $\alpha$ and $\beta$.

(6) Let $V$ be a vector space over a field $F$.

a) Let $\{v_1, ..., v_k\}$ be a set of vectors in $V$. Define the span of $\{v_1, ..., v_k\}$.

b) State what it means for $\{v_1, ..., v_k\}$ to be linearly independent.

c) State what it means for $V$ to be finite dimensional.

d) Show that if $\{v_1, ..., v_k\}$ is linearly independent but does not span $V$, then it is possible to extend it to a set $\{v_1, ..., v_{k+1}\}$ that is still linearly independent.

e) If $V$ is finite dimensional, show that by continuing as in d) you will at some point produce a basis for $V$. (You can cite results from Axler here if you wish.)